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Wound Rotor Induction Generators (WRIGs): Design and Testing

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3.1 Introduction

WRIGs have been built for powers per unit up to 400 megawatt (MW) in pump-storage power plants and down to 4.0 MW per unit in windpower plants. Diesel engine or gas-turbine-driven WRIGs for standby or autonomous operation up to 20 to 40 MW may also be practical to reduce fuel consumption and pollution for variable load.

Below 1.5 to 2 MW/unit, WRIGs are not easy to justify in terms of cost per performance against full power rating converter synchronous or cage-rotor induction generator systems.

The stator rated voltage increases with power up to 18 to 20 kV (line voltage, root mean squared [RMS]) at 400 mega voltampere (MVA). Due to limitations in voltage, for acceptable cost power converters, the rotor rated (maximum) voltage occurring at maximum slip is today about 3.5 to 4.2 kV (line voltage, RMS) with direct current (DC) voltage link alternating current (AC)–AC pulse-width modulated (PWM) converters with integrated gate controlled thyristors (IGCTs).

Higher voltage levels are approached and will be available soon for industrial use, based on multiple-level DC voltage link AC–AC converters made of insulated power cells in series and other high-voltage technologies.

So far, for the 400 MW WRIGs, the rated rotor current may be in the order of 6500 A, and thus, for $S_{max} = \pm 0.1$, approximately, it would mean 3.6 kV line voltage (RMS) in the rotor. A transformer is necessary to match the 3.6 kV static power converter to the rotor with the 18 kV power source for the stator. The rotor voltage V_r is as follows:

$$V_r = K_{rs} \cdot |S_{max}| \cdot V_s \quad (3.1)$$

For $|S_{max}| = 0.1$, $V_r = 3.6 \cdot 10^3$ V (per phase), $V_s = 18 \cdot 10^{-3} / \sqrt{3}$ V (phase): $K_{rs} = 2/1$. So, the equivalent turn ratio is decisive in the design. In this case, however, a transformer is required to connect the DC voltage link AC-AC converter to the 18 kV local power grid.

For powers in the 1.5 to 4 MW range, low stator voltages are feasible (690 V line voltage, RMS). The same voltage may be chosen as the maximum rotor voltage V_r at maximum slip.

For $S_{max} = \pm 0.25$, $V_s = 690 / \sqrt{3}$ V, $V_r = V_s$, $K_{rs} = 1/|S_{max}| = 4$. In this case, the rotor currents are significantly lower than the stator currents. No transformer to match the rotor voltage to the stator voltage is required.

Finally, for WRIGs in the 3 to 10 MW range, to be driven by diesel engines, 3000 (3600) rpm, or gas turbines, stator and rotor voltages in the 3.5 to 4.2 kV are feasible. The transformer is again avoided.

Once the stator and rotor rated voltages are settled, the design may proceed smoothly. Electromagnetic and thermomechanical designs are needed. In what follows, we will touch on mainly the electromagnetic design.

Even for electromagnetic design, we should distinguish three main operation modes:

- Generator at power grid
- Generator to autonomous load
- Brushless AC exciter (generator with rotor electric output)

The motoring mode is required in applications, such as for pump-storage power plants or even with microhydro or wind turbine prime movers.

The electromagnetic design implies a machine model, analytical, numerical, or mixed, one or more objective functions, and an optimization method with a computer program to execute it.

The optimization criteria may include the following:

- Maximum efficiency
- Minimum active material costs
- Net present worth, individual or aggregated

Deterministic, stochastic, and evolutionary optimization methods were already applied to electric machine design [1] (see also [Chapter 10](#)). Whichever the optimization method, it is useful to have sound preliminary (or general) design-geometry parameters for performance as a start for the optimization design process. This is the reason why the general electromagnetic design is our main target in what follows. Among the operation modes enlisted above, the generator at the power grid is the most frequently used, and thus, it is the one of interest here.

High- and low-voltage stators and rotors are considered to cover the entire power range of WRIGs (from 1 to 400 MW).

It is well understood that the following design methodology covers the essentials only. An industrial comprehensive design methodology is beyond our scope here.

3.2 Design Specifications — An Example

Stator rated line voltage (RMS) (Y) $V_{SN} = 0.38$ (0.46), 0.69, 4.2 (6), ..., 18 kV. Rated stator frequency $f_1 = 50$ (60) Hz. Rated ideal speed $n_{1N} = f_1/p_1 = 3000$ (3600), 1500 (1800). Maximum (ideal) speed: $n_{Max} = \frac{f_1}{p_1} (1 + |S_{max}|)$. Maximum slip $S_{max} = \pm 0.05, \pm 0.1, \pm 0.2, \pm 0.25$. Rated stator power (at unity power factor) $S_{1Ns} = 2$ MW. Rated rotor power (at unity power factor) $S_{Nr} = S_{Ns} |S_{max}| = 0.5$ MW. Rated (maximum) rotor line voltage (Y) is $V_{RN} \leq V_{SN}$. As already discussed, the stator power may go up to 350 MW (or more) with the rotor delivering maximum power (at maximum speed) of up to 40 to 50 MW and voltage $V_{Nr} \leq 4.2(6)$ kV.

Here we will consider the case of total 2.5 MW at 690 V, 50 Hz, $V_{RN} = V_{SN} = 690$ V, $S_{max} = \pm 0.25$, and rated ideal speed $n_{1N} = 50/p_1 = 1500$ rpm ($p_1 = 2$).

Electromagnetic design factors are as follows:

- Stator (core) design
- Rotor (core) design
- Stator windings design
- Rotor windings design
- Magnetization current computation

- Equivalent circuit parameters computation
- Loss and efficiency computation

3.3 Stator Design

There are two main design concepts in calculating the stator interior diameter D_{is} : the output coefficient design concept (C_e — Esson coefficient) and the shear rotor stress (f_{xt}) [1]. We will make use here of the shear rotor stress concept with $f_{xt} = 1.5 - 8 \text{ N/cm}^2$.

The shear rotor stress increases with torque. So, first, the electromagnetic torque has to be estimated, noticing that at 2.5 MW, $2p_1 = 4$ poles, the expected rated efficiency $\eta_N > 0.95$.

The total electromagnetic power S_{gN} at maximum speed and power is as follows:

$$S_{gN} \approx \frac{(S_{SN} + S_{RN})}{\eta_N} = \frac{(2 + 0.5)10^6}{0.96} = 2.604 \times 10^6 \text{ W} \quad (3.2)$$

The corresponding electromagnetic torque T_e is

$$T_e = \frac{S_{gN}}{2\pi \frac{f_1}{p_1} (1 + |S_{\max}|)} = \frac{2.604 \times 10^6}{2\pi \frac{50}{2} (1 + 0.25)} = 1.327 \times 10^4 \text{ Nm} \quad (3.3)$$

This is about the torque at stator rated power and ideal synchronous speed ($S = 0$). The stator interior diameter D_{is} , based on the shear rotor stress concept, f_{xt} is

$$D_{is} = \sqrt[3]{\frac{2T_e}{\pi \cdot \lambda \cdot f_{xt}}}; \quad \lambda = \frac{l_i}{D_{is}} \quad (3.4)$$

with l_i equal to the stack length. The stack length ratio $\lambda = 0.2$ to 1.5, in general. Smaller values correspond to a larger number of poles. With $\lambda = 1.0$ and $f_{xt} = 6 \text{ N/cm}^2$ (aiming at high torque density), the stator internal diameter D_{is} is as follows:

$$D_{is} = \sqrt[3]{\frac{2 \cdot 1.3270}{\pi \times 1 \times 6}} = 0.52 \text{ m} \quad (3.5)$$

At this power level, a unistack stator is used, together with axial air cooling.

The external stator diameter D_{out} based on the maximum airgap flux density per given magnetomotive force (mmf) is approximated in Table 3.1 [1]. So, from Table 3.1, D_{out} is as follows:

$$D_{out} = D_{is} \cdot 1.48 = 0.5200 \cdot 1.48 = 0.796 \text{ m} \quad (3.6)$$

The rated stator current at unity power factor in the stator I_{SN} is

$$I_{SN} = \frac{S_{SN}}{\sqrt{3}V_{SN}} = \frac{2 \cdot 10^6}{\sqrt{3} \cdot 690} = 1675.46 \text{ A} \quad (3.7)$$

TABLE 3.1 Outer to Inner Stator Diameter Ratio

$2p_1$	2	4	6	8	≥ 10
D_{out}/D_{is}	1.65–1.69	1.46–1.49	1.37–1.40	1.27–1.30	1.24–1.20

The airgap flux density $B_{g1} = 0.75$ T (the fundamental value). On the other hand, the airgap electro-magnetic force (emf) E_s per phase is as follows:

$$\begin{aligned} E_s &= K_E V_{SN} / \sqrt{3}; \\ K_E &= 0.97 - 0.98 \end{aligned} \quad (3.8)$$

$$E_s = \pi \sqrt{2} f_1 W_1 K_{W1a} \cdot \frac{2}{\pi} B_{g1} \cdot \tau \cdot I_i \quad (3.9)$$

where τ is the pole pitch:

$$\tau = \pi D_{is} / 2p_1 = \pi \cdot 0.52 / (2 \cdot 2) = 0.40 \text{ m} \quad (3.10)$$

K_{W1} is the total winding factor:

$$\begin{aligned} K_{W1} &= \frac{\sin \pi / 6}{q_1 \sin \pi / 6q_1} \sin \frac{\pi}{2} \frac{y}{\tau}; \\ \frac{2}{3} &\leq \frac{y}{\tau} \leq 1 \end{aligned} \quad (3.11)$$

where

q_1 is the number of slot/pole/phase in the stator

y/τ is the stator coil span/pole pitch ratio

As the stator current is rather large, we are inclined to use $a_1 = 2$ current paths in parallel in the stator. With two current paths in parallel, full symmetry of the windings with respect to stator slots may be provided. For the time being, let us adopt $K_{W1} \approx 0.910$. Consequently, from Equation 3.9, the number of turns per current path W_{1a} is as follows:

$$W_{1a} = \frac{0.97 \times 690 / \sqrt{3}}{\sqrt{2} \times 50 \times 0.910 \times 2 \times 0.75 \times 0.52 \times 0.4} = 19.32 \text{ turns} \quad (3.12)$$

Adopting $q_1 = 5$ slots/pole/phase, a two-layer winding with two conductors per coil, $n_{c1} = 2$,

$$W_{1a} = \frac{2p_1}{a_1} \cdot q_s \cdot n_{c1} = \frac{2 \cdot 2}{2} \cdot 5 \cdot 2 = 20 \quad (3.13)$$

The final number of turns/coil is $n_{c1} = 2$, $q_1 = 5$, with two symmetrical current paths in parallel. The North and South Poles constitute the paths in parallel.

Note that the division of a turn into elementary conductors in parallel with some transposition to reduce skin effects will be discussed later in this chapter.

The stator slot pitch is now computable:

$$\tau_s = \frac{\tau}{3q_1} = \frac{0.40}{3.5} = 0.0266 \text{ m} \quad (3.14)$$

The stator winding factor K_{W1} may now be recalculated if only the y/τ ratio is fixed: $y/\tau = 12/15$, very close to the optimum value, to reduce to almost zero the fifth-order stator mmf space harmonic:

$$K_{W1} = \frac{\sin \pi / 6}{5 \sin(\pi / 6 \cdot 5)} \cdot \sin \frac{\pi}{2} \cdot \frac{4}{5} = 0.9566 \cdot 0.951 = 0.9097 \quad (3.15)$$

This value is very close to the adopted one, and thus, $n_{c1} = 2$ and $q_1 = 5$, $a_1 = 2$ hold. The number of slots N_s is as follows:

$$N_s = 2p_1 q_1 m = 2 \cdot 2 \cdot 5 \cdot 3 = 60 \text{ slots} \quad (3.16)$$

As expected, the conditions to full symmetry $N_s/m_1q_1 = 60/(3 \times 2) = 10 = \text{integer}$, $2p_1/a_1 = 2 \times 2/2 = \text{integer}$ are fulfilled.

The stator conductor cross-section A_{cos} is as follows:

$$A_{cos} = \frac{I_{SN}}{a_1 \cdot j_{cos}} \quad (3.17)$$

With the design current density $j_{cos} = 6.5 \text{ A/mm}^2$, rather typical for air cooling:

$$A_{cos} = \frac{1675.46}{2 \cdot 6.5} = 128.88 \text{ mm}^2 \quad (3.18)$$

Open slots in the stator are adopted but magnetic wedges are used to reduce the airgap flux density pulsations due to slot openings.

In general, the slot width $W_s/\tau_s = 0.45$ to 0.55 . Let us adopt $W_s/\tau_s = 0.5$. The slot width W_s is

$$W_s = \left(\frac{W_s}{\tau_s} \right) \cdot \tau_s = 0.5 \cdot 0.02666 = 0.01333 \text{ m} \quad (3.19)$$

There are two coils (four turns in our case) per slot (Figure 3.1).

As we deal with a low voltage stator (690 V, line voltage RMS), the total slot filling factor, with rectangular cross-sectional conductors, may be safely considered as $K_{fills} \approx 0.55$.

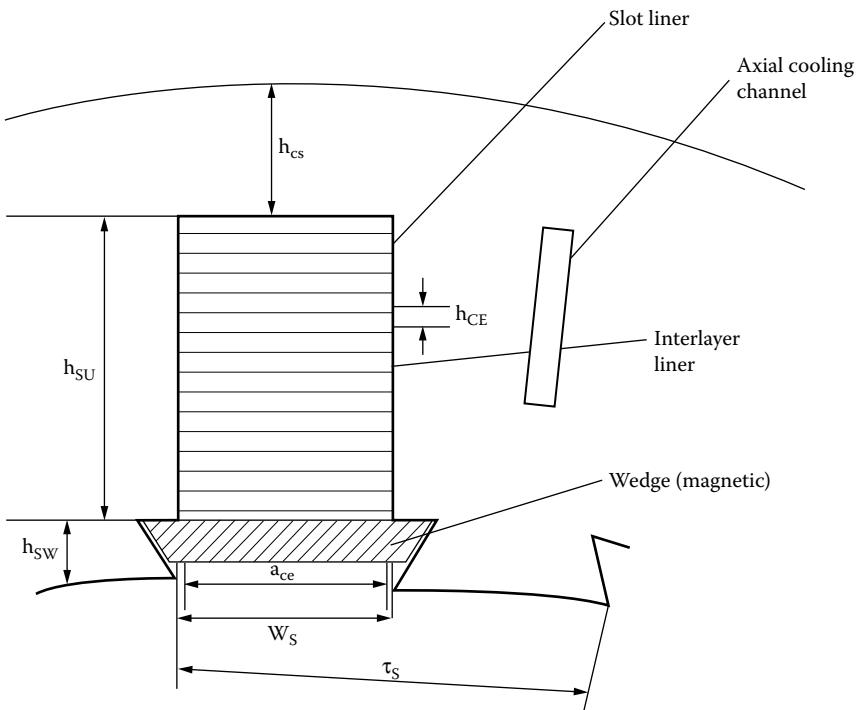


FIGURE 3.1 Stator slotting geometry.

The useful (above wedge) slot area A_{su} is

$$A_{su} = \frac{2n_{cl} A_{cos}}{K_{fills}} = \frac{2 \times 2 \times 128.88}{0.55} = 937 \text{ mm}^2 \quad (3.20)$$

The rectangular slot useful height h_{su} is straightforward:

$$h_{su} = \frac{A_{su}}{W_b} = \frac{937}{13.33} = 70.315 \text{ mm} \quad (3.21)$$

The slot aspect ratio $h_{su}/W_s = 70.315/13.33 = 5.275$ is still acceptable. The wedge height is about $h_{sw} \approx 3$ mm. Adopting a magnetic wedge leads to the apparent reduction of slot opening from $W_s = 13.33$ mm to about $W_s = 4$ mm, as detailed later in this chapter.

The airgap g is

$$g \approx (0.1 + 0.012 \sqrt[3]{S_{SN}}) 10^{-3} = (0.1 + 0.0012 \sqrt[3]{2 \cdot 10^6}) 10^{-3} = 1.612 \text{ mm} \quad (3.22)$$

With a flux density $B_{cs} = 1.55$ T in the stator back iron, the back core stator height h_{cs} (Figure 3.1) may be calculated as follows:

$$h_{cs} = \frac{B_{gl} \tau}{\pi B_{cs}} = \frac{0.75 \cdot 0.4}{\pi \cdot 1.5} = 0.0637 \text{ m} \quad (3.23)$$

The magnetically required stator outer diameter D_{outm} is

$$\begin{aligned} D_{outm} &= D_{is} + 2(h_{sm} + h_{sw} + h_{cs}) \\ &= 0.52 + 2(0.07315 + 0.003 + 0.0637) = 0.7997 \approx 0.8 \text{ m} \end{aligned} \quad (3.24)$$

This is roughly equal to the value calculated from Table 3.1 (Equation 3.6).

The stator core outer diameter may be increased by the double diameter of axial channels for ventilation, which might be added to augment the external cooling by air flowing through the fins of the cast iron frame.

As already inferred, the rectangular axial channels, placed in the upper part of stator teeth (Figure 3.1), can help improve the machine cooling once the ventilator of the shaft is able to flow part of the air through these stator axial channels.

The division of a stator conductor (turn) with a cross-section of 128.8 mm^2 is similar to the case of synchronous generator design in terms of transposition to limit the skin effects. Let us consider four elementary conductors in parallel. Their cross-sectional area A_{cose} is as follows:

$$A_{cose} = \frac{A_{cos}}{4} = \frac{128.8}{4} = 32.2 \text{ mm}^2 \quad (3.25)$$

Considering only $a_{ce} = 12$ mm, out of the 13.33 mm slot width available for the elementary stator conductor, the height of the elementary conductor h_{ce} is as follows:

$$h_{ce} = \frac{A_{cose}}{a_{ce}} = \frac{32.2}{12} \approx 2.68 \text{ mm} \quad (3.26)$$

Without transposition and neglecting coil chording the skin effect coefficient, with $m_e = 16$ layers (elementary conductors) in slot, is as follows [1]:

$$K_{Rme} = \varphi(\xi) + \frac{(m_e^2 - 1)}{3} \psi(\xi) \quad (3.27)$$

$$\begin{aligned} \xi = \beta h_{ce}; \beta &= \sqrt{\frac{\omega \mu_0 \sigma_{co}}{2} \cdot \frac{a_{ce}}{W_s}} \\ &= \sqrt{\frac{2\pi \cdot 50 \cdot 4.3 \cdot 10^7 \cdot 1.256 \cdot 10^{-6}}{2} \cdot \frac{12}{13.3}} = 0.6964 \times 10^2 m^{-1} \end{aligned} \quad (3.28)$$

Finally, $\xi = \beta h_{ce} = 0.6964 \cdot 10^{-2} \times 2.68 \cdot 10^{-3} = 0.1866$. With,

$$\varphi(\xi) = \xi \frac{(\sinh 2\xi + \sin 2\xi)}{(\cosh 2\xi - \cos 2\xi)} \approx 1.00 \quad (3.29)$$

$$\psi(\xi) = 2\xi \frac{(\sinh \xi - \sin \xi)}{(\cosh \xi + \cos \xi)} \approx 5.55 \times 10^{-4} \quad (3.30)$$

From Equation 3.27, K_{Rme} is

$$K_{Rme} = 1.00 + \frac{(16^2 - 1)}{3} \cdot 5.55 \cdot 10^{-4} = 1.04675$$

The existence of four elementary conductors (strands) in parallel leads also to circulating currents. Their effect may be translated into an additional skin effect coefficient K_{rad} [1]:

$$K_{rad} = 4\beta^4 \cdot h_{ce}^4 \left(\frac{l_i}{l_{turn}} \right)^2 n_{cn}^2 \frac{(1 + \cos \gamma)^2}{4} \quad (3.31)$$

where

l_{turn} is the coil turn length

n_{cn} is the number of turns per coil

γ is the phase shift between lower and upper layer currents ($\gamma = 0$ for diametrical coils; it is $(1 - \gamma/\tau) \frac{\pi}{2}$ for chording coils)

With $l_i/l_{turn} \approx 0.4$, $n_{cn} = 2$, $h_{ce} = 2.68 \times 10^{-3}$ m, and $\beta = 0.6964 \times 10^{-2} m^{-1}$ (from Equation 3.28), K_{rad} is as follows:

$$K_{rad} = 4 \cdot (0.6964 \cdot 10^2 \cdot 2.68 \cdot 10^{-3})^4 \cdot 0.4^2 \cdot 2^2 \frac{(1 + \cos 18^\circ)^2}{4} = 0.01847 ! \quad (3.32)$$

The total skin effect factor K_{Rll} is

$$K_{Rll} = 1 + (K_{Rme} - 1) \frac{l_{stack}}{l_{turn}} + K_{rad} = 1 + (1.04675 - 1) 0.4 + 0.01847 = 1.03717 \quad (3.33)$$

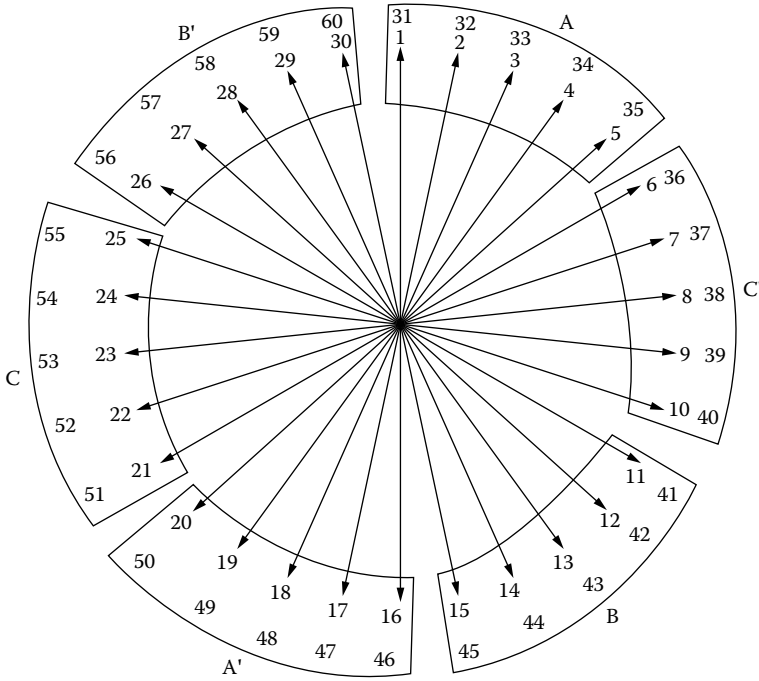


FIGURE 3.2 Slot allocation to phases for $N_s = 60$ slots, $2p_1 = 4$ poles, and $m = 3$ phases.

It seems that, at least for this design, no transposition of the four elementary conductors in parallel is required, as the total skin effect winding losses add only 3.717% to the fundamental winding losses.

The stator winding is characterized by the following:

- Four poles
- 60 slots
- $q_1 = 5$ slots/pole/phase
- Coil span/pole pitch $y/\tau = 12/15$
- $a_1 = 2$ symmetrical current paths in parallel

With t_1 equal to the largest common divisor of N_s and $p_1 = 2$, there are $N_s/t_1 = 60/2 = 30$ distinct slot emfs. Their star picture is shown in Figure 3.2. They are distributed to phases based on 120° phase shifting after choosing $N_s/2m_1 = 60/(2 \times 3) = 10$ arrows for phase a and positive direction (Figure 3.3a and Figure 3.3b).

3.4 Rotor Design

The rotor design is based on the maximum speed (negative slip)/power delivered, P_{RN} , at the corresponding voltage $V_{RN} = V_{SN}$. Besides, the WRIG is designed here for unity power factor in the stator. So, all the reactive power is provided through the rotor. Consequently, the rotor also provides for the magnetization current in the machine.

For $V_{RN} = V_{SN}$ at S_{max} , the turns ratio between rotor and stator K_{rs} is obtained:

$$K_{rs} = \frac{W_2 K_{W2}}{W_1 K_{W1}} = \frac{1}{|S_{max}|} = \frac{1}{0.25} = 4.0 \quad (3.34)$$

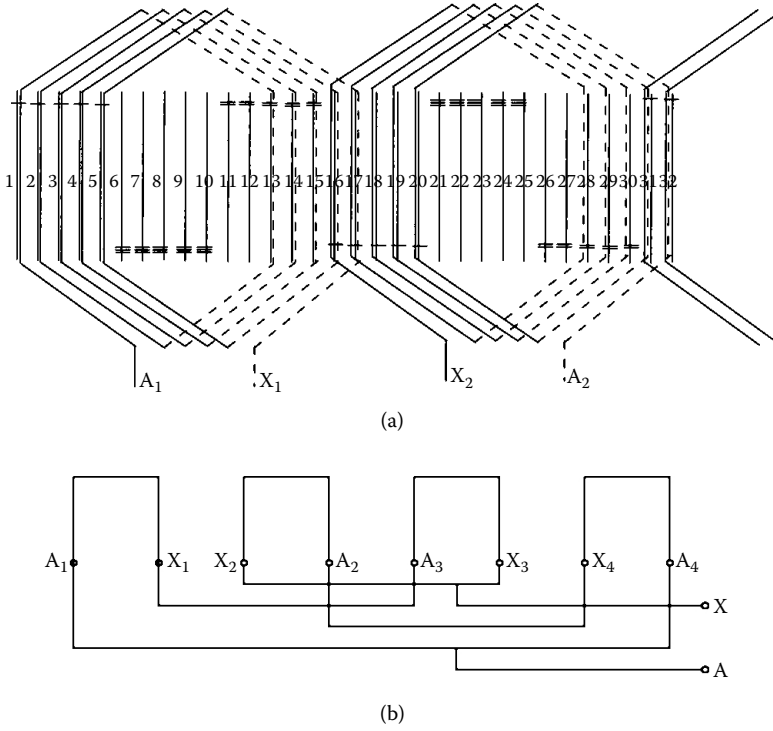


FIGURE 3.3 (a) Half of coils of stator phases A and (b) their connection to form two current paths in parallel.

Now the stator rated current reduced to the rotor I'_{SN} is as follows:

$$I'_{SN} = \frac{I_{SN}}{K_{rs}} = \frac{1675.46}{4} = 418.865 \text{ A} \quad (3.35)$$

The rated magnetization current depends on the machine power, number of poles, and so forth. At this point in the design process, we can assign I'_m (the rotor-reduced magnetization current) a per unit (P.U.) value with respect to I_{SN} :

$$I'_m = K_m I'_{SN} \quad (3.36)$$

$$K_m = 0.1 - 0.30$$

Let us consider here $K_m = 0.30$. Later on in the design, K_m will be calculated, and then adjustments will be made.

So, the rotor current at maximum slip and rated rotor and stator delivered powers is as follows:

$$I_{RN}^R = \sqrt{I_{SN}^{\prime 2} + I_m^{\prime 2}} = I'_{SN} \sqrt{1 + K_m^2} = 418.865 \sqrt{1 + 0.30^2} = 437.30 \text{ A} \quad (3.37)$$

The rotor power factor $\cos \phi_{2N}$ is

$$\cos \phi_{2N} = \frac{P_{RN}}{\sqrt{3} V_{RN} I_{RN}^R} = \frac{500000}{\sqrt{3} \cdot 690 \cdot 437.30} = 0.9578 \quad (3.38)$$

It should be noted that the oversizing of the inverter to produce unity power factor (at rated power) in the stator is not very important.

Note that, generally, the reactive power delivered by the stator Q_1 is requested from the rotor circuit as SQ_1 .

If massive reactive power delivery from the stator is required, and it is decided to be provided from the rotor-side bidirectional converter, the latter and the rotor windings have to be sized for the scope.

When the source-side power factor in the converter is unity, the whole reactive power delivered by the rotor-side converter is “produced” by the DC link capacitor, which needs to be sized for the scope.

Roughly, with $\cos\varphi_2 = 0.707$, the converter has to be oversized at 150%, while the machine may deliver almost 80% of reactive power through the stator (ideally 100%, but a part is used for machine magnetization).

Adopting $V_{RN} = V_{SN}$ at S_{max} eliminates the need for a transformer between the bidirectional converter and the local power grid at V_{SN} .

Once we have the rotor-to-stator turns ratio and the rated rotor current, the designing of the rotor becomes straightforward.

The equivalent number of rotor turns W_2K_{W2} (single current path) is as follows:

$$W_2K_{W2} = W_{1a}K_{W1} \cdot K_{rs} = 20 \times 0.908 \times 4 = 72.64 \text{ turns/phase} \quad (3.39)$$

The rotor number of slots N_R should differ from the stator one, N_S , but they should not be too different from each other.

As the number of slots per pole and phase in the stator $q_1 = 5$, for an integer q_2 , we may choose $q_2 = 4$ or 6. We choose $q_2 = 4$. So, the number of rotor slots N_R is as follows:

$$N_R = 2p_1m_1q_2 = 2 \cdot 2 \cdot 3 \cdot 4 = 48 \quad (3.40)$$

With a coil span of $Y_R/\tau = 10/12$, the rotor winding factor (no skewing) becomes

$$\begin{aligned} K_{W2} &= \frac{\sin\pi/6}{q_2 \sin\pi/6q_2} \cdot \sin\frac{\pi}{2} \cdot \frac{y_R}{\tau} = \frac{0.5}{4 \sin\frac{\pi}{24}} \cdot \sin\frac{\pi}{2} \cdot \frac{10}{12} \\ &= 0.95766 \cdot 0.9659 = 0.925 \end{aligned} \quad (3.41)$$

From Equation 3.39, with Equation 3.41, the number of rotor turns per phase W_2 is

$$W_2 = \frac{W_2K_{W2}}{K_{W2}} = \frac{72.64}{0.925} = 78.53 \approx 80 \quad (3.42)$$

The number of coils per phase is $N_R/m_1 = 48/3 = 16$. With $W_2 = 80$ turns/phase it follows that each coil will have n_{c2} turns:

$$n_{c2} = \frac{W_2}{(N_R/m_1)} = \frac{80}{16} = 5 \text{ turns/coil} \quad (3.43)$$

So, there are ten turns per slot in two layers and one current path only in the rotor.

Adopting a design current density $j_{con} = 10 \text{ A/mm}^2$ (special attention to rotor cooling is needed) and, again, a slot filling factor $K_{fill} = 0.55$, the copper conductor cross-section A_{COR} and the slot useful area A_{slotUR} are as follows:

$$A_{COR} = \frac{I_{RN}}{j_{COR}} = \frac{434.3}{10} = 43.4 \text{ mm}^2 \quad (3.44)$$

$$A_{slotUR} = \frac{2n_{c2} A_{COR}}{K_{fill}} = \frac{2 \times 5 \times 43.40}{0.55} = 789.09 \text{ mm}^2 \quad (3.45)$$

The rotor slot pitch τ_R is

$$\tau_R = \frac{\pi(D_{is} - 2g)}{N_R} = \frac{\pi(0.52 - 2 \cdot 0.001612)}{48} = 0.033805 \text{ m} \quad (3.46)$$

Assuming that the rectangular slot occupies 45% of rotor slot pitch, the slot width W_R is

$$W_R = 0.45 \tau_R = 0.45 \times 0.033805 = 0.0152 \text{ m} \quad (3.47)$$

The useful height h_{RU} (Figure 3.4) is, thus,

$$h_{SU} = \frac{A_{slotUR}}{W_r} = \frac{789.09}{15.20} = 51.913 \text{ mm} \quad (3.48)$$

This is an acceptable (Equation 3.48) value, as $h_{SU}/W_R < 4$.

Open slots have been adopted, but, with magnetic wedges ($\mu_r = 4$ to 5), the actual slot opening is reduced from $W_R = 15.2 \text{ mm}$ to about 3.5 mm , which should be reasonable (in the sense of limiting the Carter coefficient and surface and flux pulsation space harmonics core losses).

We need to verify the maximum flux density B_{tRmax} in the rotor teeth at the bottom of the slot:

$$B_{tRmax} = \frac{B_g \cdot \tau_R}{W_{tRmin}} = 0.75 \cdot \frac{33.805}{11.42} = 2.22 \text{ T} \quad (3.49)$$

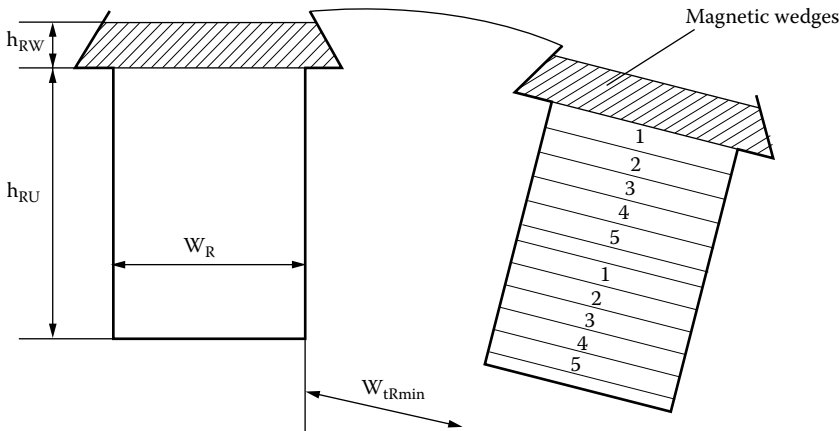


FIGURE 3.4 Rotor slotting.

with

$$\begin{aligned}
 W_{tRmin} &= \frac{\pi(D_{is} - 2(g + h_{RU} + h_{RW}))}{N_R} - W_R \\
 &= \frac{\pi(0.520 - 2(0.001612 + 0.05191) + 0.003)}{48} - 0.0152 \\
 &= 11.42 \times 10^{-3} m
 \end{aligned} \tag{3.50}$$

Though the maximum rotor teeth flux density is rather large (2.2 T), it should be acceptable, because it influences a short path length.

The rotor back iron radial depth h_{CR} is as follows:

$$h_{CR} = \frac{B_g \cdot \tau}{\pi \cdot B_{CR}} = \frac{0.75 \cdot 0.4}{\pi \cdot 1.6} = 0.0597 \approx 0.06 m \tag{3.51}$$

The value of rotor back iron flux density B_{CR} of 1.6 T (larger than in the stator back iron) was adopted, as the length of the back iron flux lines is smaller in the rotor with respect to the stator.

To see how much of it is left for the shaft diameter, calculate the magnetic back iron inner diameter D_{IR} :

$$\begin{aligned}
 D_{IR} &= D_{IS} - 2(g + h_{RU} + h_{RW} + h_{CR}) \\
 &= 0.52 - 2(0.001612 + 0.05191 + 0.003 + 0.06) = 0.287 m
 \end{aligned} \tag{3.52}$$

This inner rotor core diameter may be reduced by 20 mm (or more) to allow for axial channels — 10 mm (or more) diameter — for axial cooling, and thus, 0.267 m (or slightly less) are left of the shaft. It should be enough for the purpose, as the stack length is $l_i = D_{is} = 0.52$ m.

The rotor winding design is straightforward, with $q_2 = 4$, $2p_1 = 4$ and a single current path. The cross-section of the conductor is 43.20 mm², and thus, even a single rectangular conductor with the width $b_{cr} < W_R \approx 13$ mm and its height $h_{cr} = A_{cor}/b_{cr} = 43.40/13 = 3.338$ mm, will do.

As the maximum frequency in the rotor $f_{qmax} = f_1^* |S_{max}| = 50 \cdot 0.25 = 12.5$ Hz, the skin effect will be even smaller than in the stator (which has a similar elementary conductor size), and thus, no transposition seems to be necessary.

The winding end connections have to be tightened properly against centrifugal and electrodynamic forces by adequate nonmagnetic bandages.

The electrical rotor design also contains the slip-rings and brush system design and the shaft and bearings design. These are beyond our scope here. Though debatable, the use of magnetic wedges on the rotor also seems doable, as the maximum peripheral speed is smaller than 50 m/sec.

3.5 Magnetization Current

The rated magnetization current was previously assigned a value (30% of I_{SN}). By now, we have the complete geometry of stator and rotor slots cores, and the magnetization mmf may be considered. Let us consider it as produced from the rotor, though it would be the same if computed from the stator.

We start with the given airgap flux density $B_{g1} = 0.75$ T and assume that the magnetic wedge relative permeability $\mu_{RS} = 3$ in the stator and $\mu_{RR} = 5$ in the rotor. Ampere's law along the half of the Γ contour (Figure 3.5), for the main flux, yields the following:

$$F_m = \frac{3W_2 K_{W2} I_{R0} \sqrt{2}}{\pi p_1} = (F_{AA''} + F_{AB} + F_{BC} + F_{A'B'} + F_{B'C'}) \tag{3.53}$$

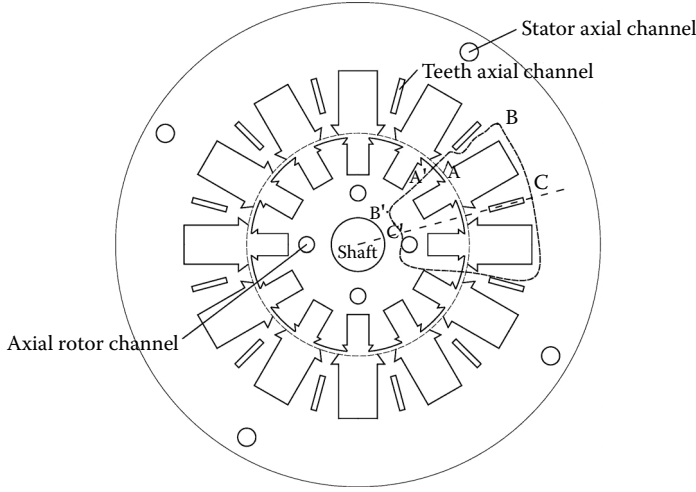


FIGURE 3.5 Main flux path.

where

$F_{AA'}$ is the airgap mmf

F_{AB} is the stator teeth mmf

F_{BC} is the stator yoke mmf

$F_{A'B'}$ is the rotor tooth mmf

$F_{B'C'}$ is the rotor yoke mmf

The airgap mmf $F_{AA'}$ is as follows:

$$F_{AA'} = gK_C \frac{B}{\mu_0}; \quad (3.54)$$

$$K_C = K_{C1} \cdot K_{C2}$$

K_C is the Carter coefficient:

$$K_{C1,2} = \frac{1}{1 - \gamma_{1,2} g / 2\tau_{s,r}} \quad (3.55)$$

$$\gamma_{1,2} = \frac{(2W_{S',R'} / g)^2}{5 + 2W_{S',R'} / g}$$

The equivalent slot openings, with magnetic wedges, $W_{S'}$ and $W_{R'}$, are as follows:

$$W_{S'} = W_S / \mu_{RS} \quad (3.56)$$

$$W_{R'} = W_R / \mu_{RR}$$

where

$$\tau_{s,r} = 26.6, 33.8 \text{ mm}$$

$$g = 1.612 \text{ mm}$$

$$W_{S'} = 13.2/3 = 4.066 \text{ mm}$$

$$W_{R'} = 15.2/5 = 3.04 \text{ mm (from Equation 3.56 through Equation 3.58)}$$

$$K_{C1} = 1.0826$$

$$K_{C2} = 1.040$$

Finally,

$$K_C = K_{C1} K_{C2} = 1.0826 \times 1.04 \approx 1.126 \quad (3.57)$$

This is a small value that will result, however, in smaller surface and flux pulsation core losses. The small effective slot opening, W_S' and W_R' will lead to larger slot leakage inductance contributions. This, in turn, will reduce the short-circuit currents.

The airgap mmf (from Equation 3.54) is as follows:

$$F_{AA'} = 1.612 \times 10^{-3} \cdot 1.126 \frac{0.75}{1.256 \times 10^{-6}} = 1083.86 \text{ Atturns}$$

The stator and rotor teeth mmf should take into consideration the trapezoidal shape of the teeth and the axial cooling channels in the stator teeth.

We might suppose that, in the stator, due to axial channels, the tooth width is constant and equal to its value at the airgap $W_{ts} = \tau_s - W_s = 26.6 - 13.2 = 13.4$ mm. So, the flux density in the stator tooth B_{ts} is

$$B_{ts} = B_{g1} \times \tau_s / W_{ts} = 0.75 \cdot \frac{0.0266}{0.0134} = 1.4888 \text{ T} \quad (3.58)$$

From the magnetization curve of silicon steel (3.5% silicon, 0.5 mm thickness, at 50 Hz; Table 3.2), $H_{ts} = 1290$ A/m by linear interpolation.

So, the stator tooth mmf F_{AB} is

$$F_{AB} = H_{ts} \cdot (h_{su} + h_{sw}) = 1290(0.070357 + 0.003) = 94.62 \text{ Atturns} \quad (3.59)$$

The stator back iron flux density was already chosen: $B_{cs} = 1.5$ T.

H_{cs} (from Table 3.2) is $H_{cs} = 1340$ A/m. The average magnetic path length l_{csw} is as follows:

$$(l_{csw})_{BC} \approx \frac{2}{3} \frac{\pi(D_{out} - h_{cs})}{2 \cdot 2p_1} = \frac{2}{3} \frac{\pi(0.780 - 0.0637)}{2 \cdot 2 \cdot 2} = 0.19266 \text{ m} \quad (3.60)$$

So, the stator back iron mmf F_{BC} is

$$F_{BC} \approx H_{cs} \cdot l_{csw} = 1340 \cdot 0.19266 \approx 258.17 \text{ Atturns} \quad (3.61)$$

TABLE 3.2 B – H Curve for Silicon (3.5%) Steel (0.5 mm thick) at 50 Hz

B (T)	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
H (A/m)	22.8	35	45	49	57	65	70	76	83	90
B (T)	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
H (A/m)	98	106	115	124	135	148	162	177	198	220
B (T)	1.05	1.1	1.15	1.2	1.25	1.3	1.35	1.4	1.45	1.5
H (A/m)	237	273	310	356	417	482	585	760	1050	1340
B (T)	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9	1.95	2.0
H (A/m)	1760	2460	3460	4800	6160	8270	11,170	15,220	22,000	34,000

In the rotor teeth, we need to obtain first an average flux density by using the top, middle, and bottom tooth values B_{trt} , B_{trm} , and B_{trb} :

$$\begin{aligned} B_{trt} &= B_{g1} \cdot \frac{\tau_R}{W_{tr}} = 0.75 \cdot \frac{0.0338}{(0.0338 - 0.0152)} = 1.3629 \text{ T} \\ B_{trm} &= B_{g1} \cdot \frac{\tau_R}{W_{trm}} = 0.75 \cdot \frac{0.0338}{0.01501} = 1.68887 \text{ T} \\ B_{trb} &= B_{g1} \cdot \frac{\tau_R}{W_{trmin}} = 0.75 \cdot \frac{0.0338}{0.01142} = 2.2 \text{ T} \end{aligned} \quad (3.62)$$

The average value of B_{tr} is as follows:

$$B_{tr} = (B_{trt} + B_{trb} + 4B_{trm}) / 6 = (1.3629 + 2.2 + 4 \times 1.68887) / 6 = 1.7197 \approx 1.72 \text{ T} \quad (3.63)$$

From Table 3.2, $H_{tr} = 5334 \text{ A/m}$. The rotor tooth mmf $F_{A'B'}$ is, thus,

$$F_{A'B'} = H_{tr} \times (h_{RU} + h_{RW}) = 5334(0.0519 + 0.003) = 292.8366 \text{ Atturns} \quad (3.64)$$

Finally, for the rotor back iron flux density $B_{CR} = 1.6 \text{ T}$ (already chosen), $H_{CR} = 2460 \text{ A/m}$, with the average path length l_{CRav} as below:

$$l_{CRav} = \frac{2}{3} \frac{\pi(D_{shaft} + h_{CR} + 0.01)}{2 \cdot 2p_1} = \frac{2}{3} \pi \frac{(0.267 + 0.01 + 0.06)}{2 \cdot 2 \cdot 2} = 0.088 \text{ m} \quad (3.65)$$

The rotor back iron mmf $F_{B'C'}$,

$$F_{B'C'} = H_{CR} \cdot l_{CRav} = 2460 \cdot 0.088 = 216.92 \text{ At} \quad (3.66)$$

The total magnetization mmf per pole F_m is as follows (Equation 3.55):

$$F_m = 1083.86 + 94.62 + 258.17 + 292.8366 + 216.92 = 1946.40 \text{ Atturns/pole}$$

It should be noted that the total stator and rotor back iron mmfs are not much different from each other. However, the stator teeth are less saturated than the other iron sections of the core. The uniform saturation of iron is a guarantee that sinusoidal airgap tooth and back iron flux densities distributions are secured. Now, from Equation 3.55, the no-load (magnetization) rotor current I_{R0} might be calculated:

$$I_{R0} = \frac{F_m \cdot \pi \cdot p_1}{3W_2 \cdot K_{W2} \cdot \sqrt{2}} = \frac{1946.4 \times \pi \cdot 2}{3 \cdot 80 \cdot 0.925 \cdot 1.41} = 39.05 \text{ A} \quad (3.67)$$

The ratio of I_{R0} to $I_{SN'}$ (Equation 3.35; stator current reduced to rotor), K_m , is, in fact,

$$K_m = \frac{I_{R0}}{I'_{SN}} = \frac{39.05}{418.865} = 0.09322 < 0.3 \quad (3.68)$$

The initial value assigned to K_m was 0.3, so the final value is smaller, leaving room for more saturation in the stator teeth by increasing the size of axial teeth cooling channels (Figure 3.1). Alternatively, the airgap may be increased up to 3 mm if so needed for mechanical reasons.

The total iron saturation factor K_s is

$$K_s = \frac{(F_{AB} + F_{BC} + F_{A'B'} + F_{B'C'})}{F_{AA'}} = \frac{(94.62 + 258.17 + 292.5366 + 216.92)}{1083.66} = 0.79568 \quad (3.69)$$

So, the iron adds to the airgap mmf 79.568% more. This will reduce the magnetization reactance X_m accordingly.

3.6 Reactances and Resistances

The main WRIG parameters are the magnetization reactance X_m , the stator and rotor resistances R_s and R_r , and leakage reactances X_{sl} and X_{rl} , reduced to the stator. The magnetization reactance expression is straightforward [1]:

$$\begin{aligned} X_m &= \omega_1 \cdot L_m; \\ L_m &= \frac{6\mu_0 (W_{la} K_{ws})^2 \tau l_i}{\pi^2 p_l g K_c (1 + K_s)} \\ L_m &= \frac{6 \cdot 1.256 \times 10^{-6} (20 \cdot 0.908)^2 \cdot 0.4 \cdot 0.52}{\pi^2 \cdot 2 \cdot 1.612 \cdot 10^{-3} \times 1.126 \times (1 + 0.79568) \cdot 2} = 8.0428 \times 10^{-3} \text{ H} \\ X_m &= 2 \cdot \pi \cdot 50 \cdot 8.0428 \cdot 10^{-3} = 2.5254 \Omega \end{aligned} \quad (3.70)$$

The base reactance $X_b = V_{SN}/I_{SN} = 690/(\sqrt{3} \cdot 1675.46) = 0.238 \Omega$. As expected, $x_m = X_m/X_b = 2.5254/0.238 = 10.610 = I_{SN}/I_{RO}$ from Equation 3.67.

The stator and rotor resistances and leakage reactances strongly depend on the end connection geometry (Figure 3.6). The end connection l_{fs} on one machine side, for the stator winding coils, is

$$\begin{aligned} l_{fs} &\approx 2(l_l + l_{l'}) + \pi h_{st} = 2 \left(l_l + \frac{\beta_s \tau}{2 \cos \alpha} \right) + \pi h_{st} \\ &= 2(0.015 + 0.8 \times 0.4 / 2 \cos 40^\circ) + \pi \cdot 0.07035 = 0.668 \text{ m} \end{aligned} \quad (3.71)$$

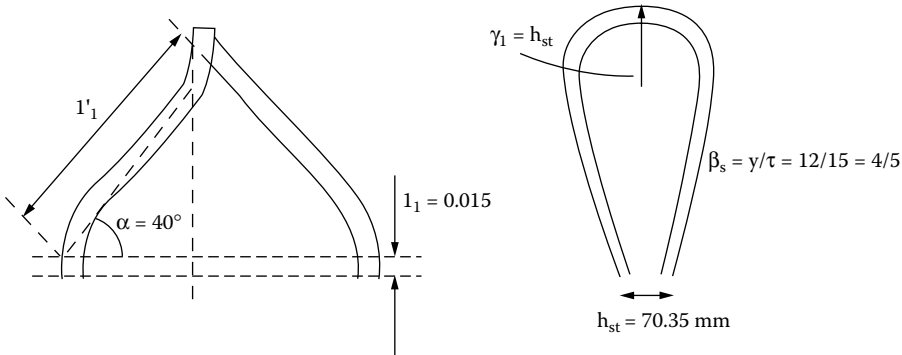


FIGURE 3.6 Stator coil end connection geometry.

The stator resistance R_s per phase has the following standard formula (skin effect is negligible as shown earlier):

$$R_s \approx \rho_{\omega_{100}^0} \cdot \frac{W_{1a} 2}{A_{\cos}} (l_i + l_{fs}) \cdot \frac{1}{a_1} \quad (3.72)$$

$$= \frac{1.8 \cdot 10^{-8}}{2} \left(1 + \frac{(100 - 20)}{272} \right) \cdot \frac{20 \cdot 2(0.52 + 0.668)}{128.88 \cdot 10^{-6}} = 0.429 \times 10^{-2} \Omega$$

The stator leakage reactance X_{sl} is written as follows:

$$X_{sl} = \omega_1 L_{sl}; L_{sl} = \mu_0 (2n_{cl})^2 \cdot l_i (\lambda_s + \lambda_{end} + \lambda_{ds}) \cdot \frac{N_s^2}{m_1 a_1^2} \quad (3.73)$$

where $n_{cl} = 2$ turns/coil, $l_i = 0.52$ m (stack length), $N_s = 60$ slots, $m_1 = 3$ phases, and $a_1 = 2$ stator current paths. λ_s , λ_{end} , and λ_{ds} are the slot, end connection, and differential geometrical permeance (nondimensional) coefficients.

For the case in point [1],

$$\lambda_s = \frac{h_{su}}{3W_s} + \frac{h_{sw}}{W'_s} = \frac{70.35}{3 \cdot 15.2} + \frac{3}{4.066} = 2.2806 \quad (3.74)$$

$$\lambda_{end} = 0.34 \cdot q_1 (l_{fs} - 0.64\beta\tau) / l_i \quad (3.75)$$

$$= 0.34 \times 5(0.668 - 0.64 \times 0.8 \times 0.4) / 0.52 = 1.5143$$

The differential geometrical permeance coefficient λ_{ds} [1] may be calculated as follows:

$$\lambda_{ds} = \frac{0.9 \cdot \tau_s (q_s K_{W1}) \cdot K_{01} \cdot \sigma_{ds}}{K_c g} \quad (3.76)$$

$$K_{01} = 1 - 0.033 \left(W_s^2 / g \tau_s \right) = 1 - 0.033 \cdot \frac{4.066^2}{1.612 \cdot 26.66} = 0.9873$$

The coefficient σ_{ds} is the ratio between the differential and magnetizing inductance and depends on q_1 and chording ratio β . For $q_1 = 5$ and $\beta = 0.8$ from Figure 3.7 [1], $\sigma_{ds} = 0.0042$.

Finally,

$$\lambda_{ds} = \frac{0.9 \cdot 0.0266(5 \cdot 0.908)^2 \cdot 0.9873 \cdot 0.0042}{1.126 \cdot 1.612 \times 10^{-3}} = 1.127$$

The term λ_{ds} generally includes the zigzag leakage flux.

Finally, from Equation 3.73,

$$L_{sl} = 1.256 \times 10^{-6} (2 \cdot 2)^2 \cdot 0.52 (2.2806 + 1.5143 + 1.127) \cdot \frac{60}{3 \cdot 2^2} = 0.257 \cdot 10^{-3} H$$

As can be seen, $L_{sl}/L_m = 0.257 \times 10^{-3} / (8.0428 \times 10^{-3}) = 0.03195$.

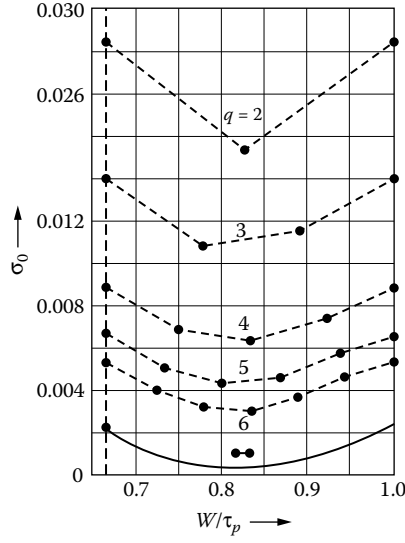


FIGURE 3.7 Differential leakage coefficient σ_d .

$X_{sl} = \omega_l L_{sl} = 2\pi \times 50 \times 0.257 \times 10^{-3} = 0.0807 \, \Omega$. The rotor end connection length l_{fr} may be computed in a similar way as in Equation 3.71:

$$l_{fr} = 2(l_i + \beta_r \tau / 2 \cos \alpha) + \pi(h_{RU} + h_{RW}) \quad (3.77)$$

$$= 2 \left(0.015 + \frac{10 \times 0.4}{12 \times 2 \times \cos 40^\circ} \right) + \pi(0.0519 + 0.003) = 0.6375 \, m$$

The rotor resistance expression is straightforward:

$$R_R^r = \rho_{\omega 100^\circ} \cdot \frac{W_2 \cdot 2}{A_{cor}} (l_i + l_{fr}) = 1.8 \cdot 10^{-8} \cdot \frac{80 \cdot 2(0.52 + 0.6775)}{43.40 \cdot 10^{-6}} = 7.68 \cdot 10^{-2} \, \Omega \quad (3.78)$$

Expression 3.73 also holds for rotor leakage inductance and reactance:

$$L_{rl}^r = \mu_0 (2n_{c2})^2 \cdot l_i (\lambda_{SR} + \lambda_{endR} + \lambda_{dR}) \cdot \frac{N_R}{m_1} \quad (3.79)$$

$$\lambda_{SR} = \frac{h_{RU}}{3W_R} + \frac{h_{RW}}{W_{R'}} = \frac{51.93}{3 \cdot 15.2} + \frac{3}{3.04} = 2.1256 \quad (3.80)$$

$$\begin{aligned} \lambda_{endR} &= 0.34 q_2 (l_{fr} - 0.64 \beta_R \tau) / l_i \\ &= 0.34 \cdot 4 \left(0.6375 - 0.64 \cdot \frac{5}{6} \cdot 0.4 \right) / 0.52 \\ &= 1.1093 \end{aligned} \quad (3.81)$$

$$\lambda_{dR} = \frac{0.9 \tau_R (q_R K_{W2})^2 \cdot K_{o2} \cdot \sigma_{dR}}{K_C g} \quad (3.82)$$

with $\sigma_{dR} = 0.0062$ ($q_2 = 4$, $\beta_R = 5/6$) from Figure 3.7:

$$K_{o2} = 1 - 0.033(W_{R'}^2)g\tau_R = 1 - 0.033(3.04^2/(1.612 \cdot 33.8)) = 0.994 \quad (3.83)$$

$$\lambda_{dR} = \frac{0.9 \cdot 33.8(4 \cdot 0.925)^2 \cdot 0.994 \cdot 0.0062}{1.126 \cdot 1.612} = 1.413$$

From Equation 3.79, L_{RL}^r is

$$L_{RL}^r = 1.256 \cdot 10^{-6} (2 \cdot 5)^2 \cdot 0.52(2.1256 + 1.1093 + 1.413) \cdot \frac{48}{3} = 4.858 \cdot 10^{-3} \text{ H}$$

$$X_{RL}^r = \omega_1 L_{RL}^r = 2\pi \cdot 50 \cdot 4.858 \cdot 10^{-3} = 1.525 \Omega$$

Noting that R_R^r , L_{RL}^r , and X_{RL}^r are values obtained prior to stator reduction, we may now reduce them to stator values with $K_{RS} = 4$ (turns ratio):

$$\begin{aligned} R_R &= R_R^r / K_{RS}^2 = \frac{7.68 \cdot 10^{-2}}{16} = 0.48 \times 10^{-2} \Omega \\ L_{RL} &= L_{RL}^r / K_{RS}^2 = \frac{4.858 \cdot 10^{-3}}{16} = 0.303 \times 10^{-3} \text{ H} \\ X_{RL} &= X_{RL}^r / K_{RS}^2 = \frac{1.525}{16} = 0.0953 \Omega \end{aligned} \quad (3.84)$$

Let us now add here the R_s , L_{sl} , X_{sl} , L_m , and X_m to have them all together: $R_s = 0.429 \times 10^{-2} \Omega$, $L_{sl} = 0.257 \times 10^{-3} \text{ H}$, $X_{sl} = 0.0807 \Omega$, $L_m = 8.0428 \times 10^{-3} \text{ H}$, and $X_m = 2.5254 \Omega$.

The rotor resistance reduced to the stator is larger than the stator resistance, mainly due to notably larger rated current density (from 6.5 A/mm² to 10 A/mm²), despite shorter end connections.

Due to smaller q_2 than q_1 , the differential leakage coefficient is larger in the rotor, which finally leads to a slightly larger rotor leakage reactance than in the stator.

The equivalent circuit may now be used to compute the power flow through the machine for generating or motoring, once the value of slip S and rotor voltage amplitude and phase are set. We leave this to the interested reader. In what follows, the design methodology, however, explores the machine losses to determine the efficiency.

3.7 Electrical Losses and Efficiency

The electrical losses are made of the following:

- Stator winding fundamental losses: p_{cos}
- Rotor winding fundamental losses: p_{cor}
- Stator fundamental core losses: p_{irons}
- Rotor fundamental core losses: p_{ironR}
- Stator surface core losses: p_{iron}^{SS}
- Rotor surface core losses: p_{iron}^{rr}
- Stator flux pulsation losses: p_{iron}^{SP}
- Rotor flux pulsation losses: p_{iron}^{RP}
- Rotor slip-ring and brush losses: p_{srb}

As the skin effect was shown small, it will be considered only by the correction coefficient $K_R = 1.037$, already calculated in Equation 3.33. For the rotor, the skin effect is neglected, as the maximum frequency $S_{max}f_1 = 0.25f_1$. In any case, it is smaller than in the stator, because the rotor slots are not as deep as the stator slots:

$$\begin{aligned} p_{cos} &= 3K_R R_S I_{SN}^2 = 3 \times 1.037 \times 0.429 \cdot 10^{-2} \cdot (1675.46)^2 \\ &= 37464.52W = 37.47 \text{ kW} \end{aligned} \quad (3.85)$$

$$\begin{aligned} p_{cor} &= 3R_R^R I_{RN}^2 = 3 \times 7.62 \cdot 10^{-2} \cdot (437.3)^2 \\ &= 43715.47W = 43.715 \text{ kW} \end{aligned} \quad (3.86)$$

The slip-ring and brush losses p_{sr} are easy to calculate if the voltage drop along them is given, say $V_{SR} \approx 1 \text{ V}$. Consequently,

$$p_{sr} = 3V_{SR} I_R^R = 3 \times 1 \times 437.3 = 1311.9 \text{ W} = 1.3119 \text{ kW} \quad (3.87)$$

To calculate the stator fundamental core losses, the stator teeth and back iron weights G_{ts} and G_{cs} are needed:

$$\begin{aligned} G_{ts} &= \left\{ \frac{\pi}{4} \left[(Dis + 2(h_{su} + h_{sw}))^2 - D_{is}^2 \right] - Ns \times (h_{su} + h_{sw}) \right\} l_i \gamma_{iron} \\ &= \left\{ \frac{\pi}{4} \left[(0.52 + 2(70.315 + 3)10^{-3})^2 \right] - 60 \times (70.315 + 3)10^{-3} \cdot 13.3 \cdot 10^{-3} \right\} \cdot 0.52 \cdot 7600 \\ &\approx 313.8 \text{ kg} \end{aligned} \quad (3.88)$$

$$\begin{aligned} G_{cs} &\approx \pi(D_{out} - h_{cs}) \times h_{cs} \times l_i \times \gamma_{iron} \\ &= \pi(0.8 - 0.0637) \times 0.0637 \cdot 0.52 \cdot 7600 \\ &= 582 \text{ kg} \end{aligned} \quad (3.89)$$

The fundamental core losses in the stator, considering the mechanical machining influence by fudge factors such as $K_t = 1.6$ to 1.8 and $K_y = 1.3$ to 1.4 , are as follows [1]:

$$P_{irons} \approx p_{10/50} \left(\frac{f_1}{50} \right)^{1.5} \left(K_t B_{ts}^2 \cdot G_{ts} + K_y B_{cs}^2 \cdot G_{cs} \right) \quad (3.90)$$

With $B_{ts} = 1.488 \text{ T}$, $B_{cs} = 1.5 \text{ T}$, $f_1 = 50 \text{ Hz}$, and $p_{10/50} = 3 \text{ W/kg}$ (losses at 1 T and 50 Hz),

$$P_{irons} = 3 \times \left(\frac{50}{50} \right)^{1.3} [1.6 \cdot (1.488)^2 \cdot 313.8 + 1.3 \cdot (1.5)^2 \cdot 582] = 8442 \text{ W} = 8.442 \text{ kW}$$

The rotor fundamental core losses may be calculated in a similar manner, but with f_1 replaced by Sf_1 , and introducing the corresponding weights and flux densities.

As the $S_{max} = 0.25$, even if the lower rotor core weights will be compensated for by the larger flux densities, the rotor fundamental iron losses would be as follows:

$$P_{ironr} < S_{max}^2 \cdot P_{irons} = \frac{1}{16} \times 8442 = 527.6 \text{ W} = 0.527 \text{ kW} \quad (3.91)$$

The surface and pulsation additional core losses, known as strayload losses, are dependent on the slot opening/airgap ratio in the stator and in the rotor [1]. In our design, magnetic wedges reduce the slot-openings-to-airgap ratio to 4.066/1.612 and 3.04/1.612; thus, the surface additional core losses are reduced. For the same reason, the stator and rotor Carter coefficients K_{c1} and K_{c2} are small ($K_{c1} \times K_{c2} = 1.126$), and therefore, the flux-pulsation additional core losses are also reduced.

Consequently, all additional losses are most probably well within the 0.5% standard value (for detailed calculations, see Reference [1], Chapter 11) of stator rated power:

$$p_{add} = p_{iron}^{SS} + p_{iron}^{SR} + p_{iron}^{SP} + p_{iron}^{RP} = \frac{0.5}{100} \cdot 2000 \cdot 10^6 = 10000 \text{ W} = 10 \text{ kW} \quad (3.92)$$

Thus, the total electrical losses Σp_e are

$$\begin{aligned} \Sigma p_e &= p_{cos} + p_{cor} + p_{irons} + p_{ironr} + p_{ad} + p_{sr} \\ &= 37.212 + 43.715 + 8.442 + 10 + 0.527 + 1.3119 = 101.20 \text{ kW} \end{aligned} \quad (3.93)$$

Neglecting the mechanical losses, the “electrical” efficiency η_e is

$$\eta_e = \frac{P_{SN} + P_{RN}}{P_{SN} + P_{RN} + \Sigma p_e} = \frac{(2+0.5)10^6}{(2+0.5)10^6 + 0.101 \cdot 10^6} = 0.9616 \quad (3.94)$$

This is not a very large value, but it was obtained with the machine size reduction in mind.

It is now possible to redo the whole design with smaller f_{xt} (rotor shear stress), and lower current densities to finally increase efficiency for larger size. The length of the stack l_i is also a key parameter to design improvements. Once the above, or similar, design methodology is computerized, then various optimization techniques may be used, based on objective functions of interest, to end up with a satisfactory design (see Reference 1, Chapter 10).

Finite element analysis (FEA) verifications of the local magnetic saturation, core losses, and torque production should be instrumental in validating optimal designs based on even advanced analytical models of the machine.

Mechanical and thermal designs are also required, and FEA may play a key role here, but this is beyond the scope of our discussion [2, 3]. Also, uncompensated magnetic radial forces have to be checked, as they tend to be larger in WRIG (due to the absence of rotor cage damping effects) [4].

3.8 Testing of WRIGs

The experimental investigation of WRIGs at the manufacturer’s or user’s sites is an indispensable tool to validate machine performance.

There are international (and national) standards that deal with the testing of general use induction machines with cage or wound rotors (International Electrotechnical Commission [IEC]-34, National Electrical Manufacturers Association [NEMA] MG1-1994 for large induction machines).

Temperature, losses, efficiency, unbalanced operation, overload capability, dielectric properties of insulation schemes, noise, surge responses, and transients (short-circuit) responses are all standardized.

We avoid a description of such tests [5] here, as space would be prohibitive, and the reader could read the standards above (and others) by himself. Therefore, only a short discussion, for guidance, will be presented here.

Testing is performed for performance assessment (losses, efficiency, endurance, noise) or for parameter estimation. The availability of rotor currents for measurements greatly facilitates the testing of WRIGs for performance and for machine parameters. On the other hand, the presence of the bidirectional power

flow static converter connected to the rotor circuits, through slip-rings and brushes, poses new problems in terms of current and flux time harmonics and losses.

The IGBT static converters introduce reduced current time harmonics, but measurements are still needed to complement digital simulation results.

In terms of parameters, the rotor and stator are characterized by single circuits with resistances and leakage inductances, besides the magnetization inductance. The latter depends heavily on the level of airgap flux, while the leakage inductances decrease slightly with their respective currents.

As for WRIGs, the stator voltage and frequency stay rather constant, and the stator flux $\bar{\Psi}_s$ varies only a little. The airgap flux $\bar{\Psi}_m$

$$\bar{\Psi}_m = \bar{\Psi}_s - L_{sl} \bar{I}_s \quad (3.95)$$

varies a little with load for unity stator power factor:

$$I_s R_s + V_s + j\omega_1 L_{sl} I_s = -j\omega_1 \bar{\Psi}_m \quad (3.96)$$

The magnetic saturation level of the main flux path does not vary much with load. So, L_m does not vary much with load. Also, unless $I_s/I_m > 2$, the leakage inductances, even with magnetic wedges on slot tops, do not vary notably with respective currents. So, parameters estimation for dealing with the fundamental behavior is greatly simplified in comparison with cage rotor induction motors with rotor skin effect.

On the other hand, the presence of time harmonics, due to the static converter of partial ratings, requires the investigation of these effects by estimating adequate machine parameters of WRIG with respect to them. Online data acquisition of stator and rotor currents and voltages is required for the scope.

The adaptation of tests intended for cage rotor induction machines to WRIGs is rather straightforward; thus, the following may all be performed for WRIGs with even better precision, because the rotor parameters and currents are directly measurable (for details see Reference [1], Chapter 22):

- Loss segregation tests
- Load testing (direct and indirect)
- Machine parameter estimation
- Noise testing methodologies

3.9 Summary

- WRIGs are built for powers in the 1.5 to 400 MW and more per unit.
- With today's static power converters for the rotor, the maximum rotor voltage may go to 4.2 to 6 kV, but the high-voltage direct current (HVDC) transmission lines techniques may extend it further for very high powers per unit.
- For stator voltages below 6 kV, the rotor voltage at maximum slip will be adapted to be equal to stator voltage; thus, no transformer is required to connect the bidirectional AC-AC converter in the rotor circuit to the local power grid. This way, the rotor current is limited around $|S_{max}| I_{SN}$; I_{SN} rated stator current.
- If stator unity power factor is desired, the magnetization current is provided in the rotor; the rotor rated current is slightly increased and so is the static converter partial rating. This oversizing is small, however (less than 10%, in general).
- For massive reactive power delivery by the stator, but for unity power factor at the converter supply-side terminals, all the reactive power has to be provided via the capacitor in the DC link, which has to be designed accordingly. As for Q_s reactive power delivered by the stator, only SQ_s has to be produced by the rotor, and the oversizing of the rotor connected static power converter still seems to be reasonable.

- The electromagnetic design of WRIGs as generators is performed at maximum power at maximum (supersynchronous) speed. The torque is about the same as that for stator rated power at synchronous speed (with DC in the rotor).
- The electromagnetic design basically includes stator and rotor core and windings design, magnetization current, circuit parameter losses, and efficiency computation.
- The sizing of WRIGs starts with the computation of stator bore diameter D_{is} , based either on Esson's coefficient or on the shear rotor stress concept $f_{xt} \approx 2.5$ to 8 N/cm^2 . The latter path was taken in this chapter. Flux densities in airgap, teeth, and back irons were assigned initial values, together with the current densities for rated currents. (The magnetization current was also assigned an initial P.U. value.)
- Based on these variables, the sizing of the stator and the rotor went smoothly. For the 2.5 MW numerical example, the skin effect in the stator and rotor were proven to be very small ($\leq 4\%$).
- The magnetization current was recalculated and found to be smaller than the initial value, so the design holds; otherwise, the design should have been redone with smaller f_{xt} .
- Rather uniform saturation of various iron parts — teeth and back cores — leads to rather sinusoidal time waveforms of flux density and, thus, to smaller core losses in a heavily saturated design. This was the case for the example in this chapter.
- The computation of resistances and leakage reactances is straightforward. It was shown that the differential leakage flux contribution (due to mmf space harmonics) should not be neglected, as it is important, though $q_1 = 5$, $q_2 = 4$ and proper coil chording is applied both in the stator and in the rotor windings.
- Open slots were adopted for both the stator and the rotor for easing the fabrication and insertion of windings in slots. However, magnetic wedges are mandatory to reduce additional (surface and flux pulsation) core losses and magnetization current.
- Though mechanical design and thermal design are crucial, they fall beyond our scope here, as they are strongly industry-knowledge dependent.
- The testing of WRIGs is standardized mainly for motors. Combining these tests with synchronous generator tests may help generate a set of comprehensive, widely accepted testing technologies for WRIGs.
- Though important, the design of a WRIG as a brushless exciter with rotor output was not pursued in this chapter, mainly due to the limited industrial use of this mode of operation.

References

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